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Hence,

$$\int_a^x \cdots \int_a^x f(x)dx^n = \frac{(x-a)^n}{\underline{|n}}\,\phi^{(n)}(a) + \frac{(x-a)^{n+1}}{\underline{|n+1}}\,\phi^{(n+1)}(a) + \frac{(x-a)^{n+2}}{\underline{|n+2}}\,\phi^{(n+2)}(a) \cdots$$

$$= \frac{(x-a)^n}{\underline{|n}} f(a) + \frac{(x-a)^{n+1}}{\underline{|n+1}} f'(a) + \frac{(x-a)^{n+2}}{\underline{|n+2}} f''(a) + \cdots.$$

Hence,

$$\frac{\underline{|n}}{(x-a)^n}\int_a^x \cdots \int_a^x f(x)dx^n = f(a) + \frac{x-a}{n+1} f'(a) + \frac{(x-a)^2}{(n+2)(n+1)} f''(a) + \cdots$$

which when n increases without limit, becomes,

$$\operatorname*{limit}_{n \doteq \infty} \frac{\underline{|n}}{(x-a)^n}\int_a^x \cdots \int_a^x f(x)dx^n = f(a).$$

425 (Calculus). Proposed by O. S. ADAMS, U. S. Coast and Geodetic Survey, Washington, D. C.

Show that the infinite product

$$(1-z)(1+\tfrac{1}{2}z)(1-\tfrac{1}{3}z)(1+\tfrac{1}{4}z)\cdots = \frac{\sqrt{\pi}}{\Gamma(1+\frac{1}{2}z)(\frac{1}{2}-\frac{1}{2}z)}.$$

SOLUTION BY C. F. GUMMER, Kingston, Ont.

The right member should evidently have been printed

$$\frac{\sqrt{\pi}}{\Gamma(1+\frac{1}{2}z)\Gamma(\frac{1}{2}-\frac{1}{2}z)},$$

since this equals

$$\frac{\Gamma(\frac{1}{2})}{\Gamma(1+\frac{1}{2}z)\Gamma(\frac{1}{2}-\frac{1}{2}z)},$$

$$= \operatorname*{Lim}_{n\to\infty}\left[\left(\frac{n^{\frac{1}{2}}}{\frac{1}{2}}\prod_{i=1}^{n}\frac{i}{i+\frac{1}{2}}\right)\left(\frac{1+\frac{1}{2}z}{n^{1+\frac{1}{2}z}}\prod_{i=1}^{n}\frac{i+1+\frac{1}{2}z}{i}\right)\left(\frac{\frac{1}{2}-\frac{1}{2}z}{n^{\frac{1}{2}-\frac{1}{2}z}}\prod_{i=1}^{n}\frac{i+\frac{1}{2}-\frac{1}{2}z}{i}\right)\right]$$

$$= \operatorname*{Lim}_{n\to\infty}\left[\frac{(1-z)(1+\frac{1}{2}z)}{n}\prod_{i=1}^{n}\left\{\left(1-\frac{z}{2i+1}\right)\left(1+\frac{z}{2i+2}\right)\frac{i+1}{i}\right\}\right]$$

$$= \operatorname*{Lim}_{n\to\infty}\left[(1-z)(1+\tfrac{1}{2}z)\prod_{i=1}^{n}\left\{\left(1-\frac{z}{2i+1}\right)\left(1+\frac{z}{2i+2}\right)\right\}\right]$$

$$= \prod_{i=0}^{\infty}\left\{\left(1-\frac{z}{2i+1}\right)\left(1+\frac{z}{2i+2}\right)\right\},$$

as required.

440 (Calculus). Proposed by J. B. REYNOLDS, Lehigh University.

If t is the differential vector joining two consecutive points on a space curve, and R is the radius of curvature at that point prove that

$$R^2 = \frac{(t\cdot t)^3}{(t\times \dot{t})\cdot(t\times\dot{t})}.$$

I. SOLUTION BY LOUIS WEISNER, Student, College of the City of New York.

Let $r = f(\tau)$ be the vector equation of the curve, s its length measured from a fixed point, and K the directed curvature.

Then,

$$K = \frac{d^2r}{ds^2} = \frac{\dfrac{d}{d\tau}\left[\dfrac{\dfrac{dr}{d\tau}}{\dfrac{ds}{d\tau}}\right]}{\dfrac{ds}{d\tau}} = \frac{\dfrac{d}{d\tau}\left[\dfrac{t}{\dfrac{ds}{d\tau}}\right]}{\dfrac{ds}{d\tau}},$$